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Beyond (+) and (-): Investigating prospective elementary school teachers' misconceptions in mathematical operations

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Abstract.
This study investigates prospective elementary school teachers' misconceptions of the (+) and (−) symbols by exploring their perceptions in various mathematical contexts. The aim is to understand how students interpret these fundamental operations and identify potential challenges in their conceptualization. Since teachers' understanding of mathematical symbols plays a crucial role in shaping students' learning experiences, it is essential to uncover gaps between their conceptual and procedural knowledge. This study involved 65 undergraduate students majoring in elementary teacher education, selected to ensure a diverse range of academic backgrounds and experiences. Data were collected through written assessments and semi-structured interviews, then analyzed thematically to identify recurring misconceptions. The findings indicate that while students demonstrate procedural fluency in basic arithmetic, significant misunderstandings arise when dealing with negative numbers, inverse operations, and algebraic expressions. Many students perceive the minus sign solely as an operator rather than a representation of a negative value, leading to difficulties in interpreting mathematical expressions in different contexts. These results highlight the need for instructional approaches that emphasize conceptual understanding alongside procedural skills. The study contributes to mathematics education research by providing insights into how prospective teachers interpret mathematical symbols and offering recommendations for improving teacher preparation programs.

Keywords:
Addition; subtraction; arithmetic operations; symbolic interpretation; misconceptions

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INTRODUCTION

The mastery of fundamental mathematical concepts plays a crucial role in shaping future elementary school teachers' ability to foster meaningful mathematics learning. Among these fundamental concepts, addition (+) and subtraction (−) symbols are often introduced early but are commonly misunderstood. Research has shown that many students and even prospective teachers tend to perceive mathematical symbols merely as operational commands rather than as carriers of conceptual meaning (Tall, 1991; Behr et al., 1992). Misinterpretations of symbols such as (+) and (−) can have far-reaching effects, impacting not only computational skills but also students' broader mathematical reasoning.

Prior studies have revealed that procedural learning often dominates over conceptual understanding in mathematics instruction (Sfard, 1991; Carpenter et al., 1999). This imbalance leads to fragile knowledge structures, particularly in the use and interpretation of mathematical

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symbols. As Vlassis (2004) notes, the minus sign (−) can represent a binary operation (subtraction), a unary operation (negative number), or a relational comparison (difference), yet many learners are unable to navigate these multiple meanings effectively. When these ambiguities are not properly addressed, students may develop persistent misconceptions that hinder their ability to engage with more complex mathematical ideas.

While considerable attention has been given to children's misconceptions regarding basic operations, relatively few studies have explored the conceptual understanding of prospective elementary school teachers. Given their future role in shaping early mathematical experiences, it is critical to investigate how these teachers comprehend fundamental symbols and operations. Without a solid conceptual foundation, prospective teachers may inadvertently propagate misunderstandings among their students, perpetuating cycles of mathematical difficulties (Khalid & Embong, 2020). Thus, this study aims to explore prospective elementary school teachers' understanding of addition and subtraction symbols across various mathematical contexts. By identifying common misconceptions and conceptual gaps, this research seeks to contribute to the development of more effective teacher education strategies that promote deep mathematical understanding.

Mathematical symbols, particularly addition (+) and subtraction (−), are not mere operational signs but carry complex conceptual meanings. Tall (1991) emphasized that symbols in mathematics act as both processes and objects, indicating that (+) and (−) must be understood beyond simple procedures. Similarly, Wüstenberg et al. (2012) illustrated that a deep understanding of the symbol (−) is necessary not only to perform subtraction but also to correctly interpret negative numbers and differences between quantities.

Misunderstandings related to the symbols (+) and (−) are well-documented in research. Behr et al. (1992) found that many students interpret the minus symbol solely as a subtraction command, neglecting its use in indicating negative quantities. This procedural bias often leads to significant difficulties when students engage with operations involving negative numbers or algebraic structures. Supporting this view, Siegler and Lortie-Forgues (2015) revealed frequent student misinterpretations of the commutative properties involving subtraction, further indicating a lack of conceptual grounding. Additionally, Khalid and Embong (2020) noted that preservice teachers often misinterpret the minus sign, causing challenges in higher-level algebraic manipulation and understanding inverse operations. Rittle-Johnson & Schneider, (2015) stressed that a misunderstanding of the relational meaning between addition and subtraction can hinder students' progress towards more abstract mathematical thinking.

Although there are substantial literature documenting students' misconceptions of mathematical symbols, research focusing specifically on prospective elementary school teachers remains relatively limited. For instance, Bofferding (2014) focused on young learners' understanding of negative numbers, revealing early misconceptions, but similar studies on preservice teachers are scarce. Studies such as Tirosh and Stavy (1999) examined intuitive rules in mathematical reasoning but did not directly address teaching practices among preservice elementary teachers.

Recent studies highlight that traditional instruction tends to prioritize procedural fluency over conceptual understanding (e.g., Behr et al., 1992; Sfard, 1991). This suggests that teacher education programs may need more targeted interventions that help preservice teachers build robust conceptual models of basic operations like addition and subtraction across different mathematical contexts. Existing research has primarily focused on simple operations with young students or general student populations. There is a gap regarding how preservice elementary teachers, who will become future instructors, conceptualize the meanings of (+) and (−) beyond elementary procedures. Specifically, there is a lack of research exploring how these future teachers negotiate the multiple contexts in which (+) and (−) operate (e.g., negative numbers, algebraic expressions, directional change). Furthermore, few studies have systematically investigated the link between conceptual and procedural knowledge in preservice teachers' understanding of these operations. Thus, this study aims to contribute by explicitly investigating the conceptual frameworks and misconceptions that preservice elementary school teachers have regarding the symbols (+) and (−), within a range of mathematical contexts.

METHOD

This study employed a qualitative approach using a case study design to investigate misconceptions among prospective elementary school teachers in interpreting the addition (+) and subtraction (−) symbols in various mathematical contexts. Although initially designed as a phenomenological study, the primary focus on written assessments and semi-structured interviews, without an in-depth exploration of participants' lived experiences, aligns more closely with a case study approach.

The participants consisted of 65 students enrolled in an elementary school teacher education program, selected through purposive sampling to ensure diversity in academic profiles. The diversity criteria considered included participants' Grade Point Average (GPA), current semester, and prior teaching or tutoring experiences. These criteria were chosen to capture a range of understandings shaped by different levels of academic achievement and exposure to classroom or instructional settings.

The research instruments consisted of a written diagnostic test and semi-structured interview guidelines. The written test was developed to assess students' conceptual and procedural understanding of addition and subtraction symbols across multiple mathematical contexts. As presented in Table 1, the test includes three main components: (1) *Conceptual Meaning* — assessing students' understanding of the fundamental meaning of the symbols (+) and (−) in arithmetic and algebraic expressions; (2) *Understanding Symbol in Different Contexts* — evaluating how students interpret the symbols in various scenarios such as word problems, number lines, and algebraic transformations; and (3) *Connection Between Concepts* — identifying students' ability to connect symbolic meaning with conceptual reasoning, including justifying or refuting mathematical statements. To maintain instrument validity, the test items and interview questions were developed based on relevant literature and reviewed by two mathematics education experts. A pilot test was conducted with 10 non-participant students from a different institution to check clarity and consistency. Triangulation between written responses and interview data was used to enhance data credibility.

Table 1. Grid of questions on the written test

Component	Questions
Conceptual Meaning	What does the (+) and (-) symbols mean? a) $15 + (-7)$ b) $-12 - (-6)$ c) $-(x + 2)$
Understanding Symbol in Different Context	Explain the different contexts of using the (+) and (-) symbols in the following operations: a) $10 - (-4)$ b) $7 + (-3)$ c) $-x - 2$
Connection Between Concept	Is it true or false? Give the reasons. a) $(-2) - (-6) + (-4) = (-4) + (-6) - (-2)$ b) $-(x - 2) = -x - 2$

Data collection involved administering the written test, followed by semi-structured interviews with a selected subset of participants representing different error patterns. Interviews aimed to probe deeper into the reasoning behind their written responses and uncover underlying misconceptions. Data analysis was conducted thematically through a phenomenologically-informed lens, focusing on recurring patterns and categories that emerged from participants' explanations. The analysis followed steps adapted from phenomenological thematic procedures, including: a) Bracketing–setting aside researchers' prior assumptions to remain open to participants' perspectives; b) Horizonalization–identifying significant statements in written and verbal data; c) Clustering into Themes–grouping meaning units into broader themes such as misinterpretation of symbols, procedural dependence, or relational misunderstanding; and d) Textural and Structural Descriptions–constructing descriptions to represent both what participants understood and how they experienced the concept.

The researchers also acknowledged their role as the primary instruments in data interpretation. Reflexive notes were maintained during analysis to account for researcher subjectivity, and member checking was carried out with selected participants to validate interpretations of their responses. This multi-method approach aimed to produce an in-depth understanding of how elementary teacher candidates conceptualize fundamental mathematical operations, providing insights into potential gaps in their pedagogical content knowledge.

RESULTS AND DISCUSSION

The findings revealed that many prospective elementary school teachers (PSTs) struggled with basic arithmetic operations, particularly when negative numbers and parentheses were involved. One of the most recurrent errors was the misinterpretation of expressions such as $-(x+2)$ as $-x+2$. This indicates a fundamental misunderstanding of the hierarchical structure of operations and the role of parentheses, which reflects a conceptual misunderstanding rather than a simple computational slip.

Conceptual Misunderstanding of Negative Signs and Parentheses

Participants' written work and follow-up interviews revealed that the negative sign was often perceived as a static symbol rather than as an operator affecting the entire expression. This finding echoes Behr et al. (1992), who emphasized the role of symbol interpretation in developing mathematical meaning. However, unlike Behr et al., this study employed a phenomenological lens to explore how these misunderstandings are rooted in the *lived experiences* of PSTs—such as how they were introduced to operations in earlier schooling. For instance, one participant explained: "*We usually just cancel the negative sign or flip the sign inside the bracket, because that's how I learned it in school*."

This comment illustrates how rote procedures override conceptual reasoning, supporting Mulligan and Mitchelmore's (2009) view on fragmented early mathematical schemas. Vlassis (2004, 2008) further noted that the negative sign is one of the most misunderstood symbols in elementary mathematics, often reduced to a signal for "opposite" without proper operational meaning. Our findings corroborate and extend Vlassis's claims in 2008 by showing that the PSTs' misconception is not only symbol-based but also deeply connected to how operations are mentally represented and interpreted contextually.

Contextual Influences on Misconception

Several errors emerged when participants were given word problems involving subtraction or loss, especially in contexts that involved money or temperature. In one case, a participant incorrectly evaluated "*You have $5 and spend $8*" as "$+3$ rather than -3", revealing a failure to map real-world experiences onto symbolic expressions. While they understood the context of "loss", they failed to encode it mathematically (Larson & Edwards, 2015).

The interviews suggested that this issue stemmed from the absence of mental models or visual representations that link symbolic operations to real-life actions. This aligns with Van de Walle's (2018) emphasis on the need for meaningful models such as number lines. However, rather than merely recommending the use of number lines generically, we found that when participants were asked to construct number lines themselves during the interviews, their reasoning improved. For example, one participant revised their answer after drawing a number line from 5 to -3, stating: "*I can see now I went back more than I had, so it has to be negative.*" This finding highlights the transformative potential of self-generated representations, a strategy more aligned with constructivist learning than with teacher-led demonstrations.

Conceptual Connections: Gaps in Structural Understanding

Another recurring error was observed in manipulating expressions involving distribution, such as interpreting $-(x+2)$ incorrectly as $-x+2$. Interestingly, this was discussed by the same participant in both the Conceptual and Connecting sections, suggesting a disintegration between arithmetic and algebraic thinking (Bittinger, et al., 2017). While participants could perform basic arithmetic, their failure to apply distributive properties correctly suggests a lack of structural understanding of expressions (Kieran, 2007).

The phenomenological approach further revealed that participants tended to compartmentalize arithmetic and algebra as distinct learning experiences. They described learning algebraic expressions as a separate topic in secondary school, often detached from the operations they had previously learned. This lack of continuity contributes to the failure to transfer conceptual understanding across domains.

Integrating Written and Interview Data

The convergence between written test data and interviews provided a richer understanding of these errors. For example, participants who made the same written mistake—such as misapplying order of operations—articulated vastly different justifications during the interviews. One participant believed the calculator dictated operation order, while another cited a mnemonic (PEMDAS) but misapplied it. These discrepancies highlight the need to analyze beyond written work and attend to participants' justifications and mental models, which phenomenology enables.

Implications for Teaching and Learning

Recommendations emerging from this study are grounded in the actual experiences and thought processes of PSTs, rather than generic pedagogical advice. For instance:

- To address the $-(x+2)$ error, explicit instruction on the distributive property should be accompanied by counterexamples and "think-aloud" tasks, where students predict and justify multiple interpretations.
- Instead of merely introducing number lines, students should be asked to create and explain them, linking symbolic expressions with contextual actions.
- Classroom discussions should include reflective prompts, such as: "What does this symbol mean to you?" or "What does the negative sign *do* to this expression?"

These approaches offer pedagogical strategies that are responsive to the specific misconceptions uncovered, and not simply reiterations of general recommendations found in textbooks (Novak & Gowin, 1984).

Contribution to the Literature and Further Research

This study contributes to the literature by bridging symbolic misconceptions and subjective experiences of learning, an area underexplored in prior studies. While Vlassis (2004) and Stein et al. (2005) have highlighted symbolic ambiguity, our findings provide a phenomenological account of how these ambiguities are internalized, normalized, and carried forward into prospective teachers' cognition.

Future research should investigate how these early symbol interpretations evolve through teacher education programs and how reflective teaching practices might reframe prior misconceptions. This would not only deepen conceptual understanding but also equip future teachers to anticipate and address these issues in their own classrooms.

CONCLUSIONS

This study uncovered persistent misconceptions among prospective elementary school teachers (PSTs) in understanding and performing basic mathematical operations, particularly those involving negative numbers, parentheses, and algebraic expressions. Through a phenomenological lens, the research revealed that these errors are not merely procedural but are rooted in fragmented conceptual understanding, disconnected learning experiences, and symbolic misinterpretations that have been internalized over time.

The originality of this study lies in its integration of lived experiences with cognitive analysis, shedding light on how PSTs make sense of mathematical symbols such as the negative sign and parentheses. Unlike previous studies that often remain at the surface of written errors, this research connects the dots between how symbols are introduced, experienced, and carried forward into teacher cognition.

Pedagogically, the findings point to the need for deep representational learning, reflective practices, and instructional strategies that address specific misconceptions. Tools such as self-constructed number lines, counterexamples, and student-led explanation tasks can bridge the gap between symbolic manipulation and conceptual meaning. These strategies, grounded in the participants' actual reasoning, offer more targeted interventions than general recommendations often found in the literature.

Future research should explore longitudinal trajectories of symbol interpretation from early schooling through teacher education programs, with an emphasis on how reflective dialogue and meta-cognitive awareness can help PSTs reconstruct their understanding. Additionally, further studies could investigate how these symbolic misconceptions influence teaching practices and the transmission of errors to the next generation of learners.

By re-centering the discussion around the subjective mathematical experiences of PSTs, this study emphasizes the importance of understanding not only *what* errors occur, but *why* they persist—and how teacher education must address both content knowledge and the conceptual histories that shape it.

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