

Fraction or division? Conceptual misunderstandings in early fraction instruction in elementary school

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Abstract.

Many elementary students encounter difficulties in comprehending fractions conceptually and frequently confuse them with basic division operations. This persistent misconception is partially attributed to the manner in which fractions are introduced in early-grade textbooks, which often employ simplified narratives and visual contexts—such as dividing apples or coloring shapes—that obscure the distinction between “a part of a whole” and “a result of division.” This study investigated the impact of such textbook content and classroom practices on these misunderstandings. Employing a qualitative research approach, the study involved two second-grade teachers and 55 students (aged 7–8 years) and integrated content analysis of a widely utilized second-grade mathematics textbook, diagnostic assessments, and interviews. The analysis revealed that fractions are frequently depicted as division outcomes (e.g., $1:4 = 1/4$), thereby reinforcing procedural thinking and impeding students’ capacity to grasp the abstract nature of fractions. The study concludes that effective early fraction instruction necessitates the incorporation of diverse representations and explicit differentiation between fractions and division. It is recommended that educators utilize multiple representations and precise mathematical language to facilitate students’ conceptual comprehension.

Keywords:

Conceptual
misconceptions; elementary
mathematics; fraction
understanding; textbook
analysis; visual
representation

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INTRODUCTION

Comprehending the concept of fractions presents a significant challenge in the elementary mathematics curriculum. Numerous students encounter difficulties in grasping the conceptual understanding of fractions, often equating them with basic division operations (Kalra et al., 2020; Lee & Boyadzhiev, 2020; Ramadianti et al., 2026). This misconception is not solely attributable to the limitations of students’ cognitive capacities, but can also be attributed to the manner in which the concept of fractions is introduced in textbooks and classroom teaching practices (Aliustaoğlu et al., 2018; Baştürk, 2024; Braithwaite & Siegler, 2023; Adelia et al., 2025).

In theory, fractions are defined as representations of parts of a whole, not merely the outcome of a division operation (Lamon, 2012). However, research indicates that students develop meanings of fractions through their lived experiences and interpretations, which are frequently influenced by instructional contexts and representations (Isnawan et al., 2022). Nevertheless, in practice, numerous textbooks and educators in the field introduce fractions in a manner that

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simplifies the concept to “divided by so much,” such as dividing an apple into four parts to represent $\frac{1}{4}$. This type of visual and narrative representation can lead to confusion among students, resulting in the assumption that all fractions are equivalent to the result of a division operation (e.g., $1:4 = \frac{1}{4}$) (Pitta-Pantazi et al., 2020).

The disparity between theoretically proposed conceptual approaches and the procedural approaches employed in the field indicates an urgent need to re-evaluate the early teaching of fraction concepts. Several previous studies have emphasized the significance of employing multiple representations, including area, length, and set models, to foster a more robust conceptual understanding of fractions (Ardiansari et al., 2024; Park & Esposito, 2024; Mulligan & Mitchelmore, 2016). Regrettably, such practices are still infrequently encountered in textbooks and classroom instruction at the early elementary school level, particularly in Indonesia.

This study diverges from this phenomenon and seeks to investigate how the content presented in grade 2 elementary school mathematics textbooks, as well as classroom teaching practices, contribute to students’ misconceptions regarding the concept of fractions. Consequently, this study is anticipated to offer theoretical and practical contributions in the development of more conceptual teaching materials and pedagogical approaches, thereby facilitating students’ acquisition of a deeper and more adaptable comprehension of the concept of fractions from an early age.

This study presents a novel contribution by integrating textbook content analysis with classroom practice analysis to systematically trace the origins of students’ misconceptions regarding fractions. Unlike previous studies that focus either on students’ errors or instructional materials separately, this study provides a comprehensive perspective by connecting representations in textbooks, teachers’ instructional language, and students’ conceptual interpretations within the Indonesian context. Accordingly, this study addresses the following research questions:

1. How are fractions conceptualized and procedurally presented in grade 2 mathematics textbooks?
2. How do classroom teaching practices impact students’ comprehension of fractions?
3. What are the common misconceptions that students exhibit when interpreting fractions?
4. How do textbook representations and teaching practices contribute to these misconceptions?

METHOD

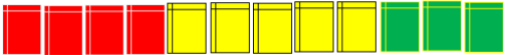
This study employed a descriptive qualitative approach to investigate the influence of textbook content and classroom practices on elementary school students’ conceptual misconceptions in comprehending fractions. This methodology was selected due to its suitability for elucidating students’ understanding in depth through the analysis of non-numerical data, such as narratives in textbooks and student responses (Creswell & Creswell, 2017). The research design integrated content analysis of a widely utilized second-grade mathematics textbook in Indonesia, diagnostic assessments, and interviews. The participants comprised 55 second-grade students (aged 7–8 years) from a public elementary school in Probolinggo, East Java, Indonesia. The students originated from relatively homogeneous socio-economic backgrounds and had previously been introduced to fundamental fraction concepts through their regular mathematics curriculum. Furthermore, two second-grade teachers with over five years of teaching experience participated in this study. Both teachers were responsible for teaching mathematics and were acquainted with the textbook analyzed in this research. Textbooks were selected because they serve as the primary source in introducing fraction concepts at an early stage (Andriatna, 2025).

The diagnostic test was meticulously developed by the researchers, drawing upon pivotal conceptual indicators of fraction comprehension. These indicators encompass part-whole relationships, symbolic representation, and the comparative analysis of fractions. To ensure content validity, the test underwent expert judgment by two mathematics education specialists.

Reliability was evaluated through a pilot test conducted with a select group of students. Ambiguous items were subsequently revised to enhance clarity and consistency.

Data collection was conducted through three primary techniques: a) content analysis of textbooks, focusing on visual and narrative representations of fractions, particularly their introduction methods (e.g., dividing an apple or coloring shapes). The coding instrument was structured based on categories of fraction representation: model area, model set, length, and type of conceptual or procedural explanation (Watt et al., 2025); b) semi-structured interviews with two teachers and five students to explore teachers' understanding of their fraction teaching methods and students' comprehension of the concept of fractions and division operations. Questions were designed to identify students' tendencies to equate fractions with quotients (Lamon, 2007); c) diagnostic tests (refer to Table 1) to determine whether students comprehend fractions as parts of a whole or merely as the outcome of division. The combination of content analysis, diagnostic tests, and interviews was selected to facilitate methodological triangulation. Content analysis enabled the identification of how fractions are conceptually presented in textbooks, diagnostic tests captured students' observable misconceptions, and interviews provided deeper insights into students' reasoning and teachers' instructional practices. This combination ensured a comprehensive understanding of the phenomenon from multiple perspectives.

Table 1. Diagnostic test questions

Questions	Focus
In the fraction $\frac{3}{5}$, the number 5 represents: a. The number of parts owned b. The number of cakes c. The total number of parts d. The remaining part	The meaning of the denominator in fractions
Try writing with the following fraction words: a. $\frac{1}{2}$; b. $\frac{3}{4}$; c. $\frac{2}{3}$	Fractional language and its pronunciation
There is a cake divided into 4 equal parts. Ali gets 1 part. Write a fraction that shows the part that Ali gets!	The concept of the denominator indicates the total number of parts, not the parts owned
If you have $\frac{1}{4}$ of a cake and your friend has $\frac{1}{2}$ of a cake, who has more cake? a. Me b. My friend c. Equally many d. Don't know	Comparing fractions conceptually
Mother has 12 books, consisting of:  All the red books are given to Ali. How many of the total books does Ali receive? a. 1:3 b. $\frac{1}{3}$ c. 12:4 d. $\frac{4}{12}$	The concept between division and fraction symbols

Data analysis was conducted using thematic content analysis techniques (Braun & Clarke, 2006), which were implemented in two phases. Initially, the textbook analysis results were categorized based on representation and explanation. Subsequently, the results of observations, interviews, and tests were analyzed thematically to identify patterns of misconceptions, such as the tendency of students to assert that " $1:4 = \frac{1}{4}$ " without fully comprehending the concept of fractions. Data triangulation was employed by comparing the results of the three data collection

methods to enhance the validity of the findings (Sharp, 2003). These findings were subsequently compared with research-based pedagogical approaches that advocate for the utilization of multiple representations in fraction learning (Björklund et al., 2020).

This study utilized the praxeological framework derived from the Anthropological Theory of Didactics by Chevallard, which analyzes mathematical activity through four components: types of tasks (T), techniques (τ), technologies (θ), and theories (Θ). This framework was employed to investigate how fraction concepts are presented in textbooks and enacted in classroom practices, specifically whether the emphasis is placed on procedural techniques (e.g., dividing objects) or conceptual technologies (e.g., comprehending part-whole relationships).

RESULTS AND DISCUSSION

Comprehending Fraction Denominators (Question 1)

Historically, fractions have been comprehended through various interpretations, encompassing part-whole, quotient, ratio, operator, and measure perspectives (Kieren, 1980; Lamon, 2007). Notably, the part-whole interpretation is predominantly introduced during early elementary education. However, an excessive emphasis on the quotient interpretation (fractions as division) without explicit conceptual clarification can result in enduring misconceptions.

Only 21 students (38%) of the 55 students answered correctly, specifically stating “the total number of parts.” A significant number of students, 20 (36%), incorrectly identified “the number of parts owned.” The remaining students provided alternative answers, such as “the number of cakes” or “the remaining part.” This finding suggests that a substantial portion of students lack a clear understanding of the denominator’s role as an indicator of the number of parts within a whole unit. They tend to perceive the number beneath the fraction (denominator) as the portion they possess or mistakenly believe that the numerator and denominator share the same meaning. For instance, one student (S12) expressed confusion by stating that in the fraction $3/5$, “5 represents the quantity we possess,” indicating a misunderstanding of the relationship between the numerator and denominator. Another student (S27) clarified that “5 signifies the number of parts we consume,” demonstrating a reversal of conceptual roles.

This aligns with Lamon’s (2007) findings, which indicate that elementary school students frequently interpret fractions as “certain parts owned” without associating them with the concept of a whole that is divided equally. Consequently, the full development of the meaning of fractions as proportional representations is hindered. This misconception is a prevalent type of conceptual error, wherein students are unable to discern the distinct roles of the numerator and denominator.

The denominator of a fraction serves as a numerical representation of the number of equal parts into which the whole is divided. Derived from the Latin word “denominare,” meaning “to name,” the denominator provides information about the size of each part. For instance, in the fraction $\frac{1}{2}$, each part is referred to as a half, and two halves constitute a whole. Similarly, in the fraction $\frac{1}{3}$, each part is called a third, and three thirds make up a whole. In the fraction $\frac{1}{4}$, each part is known as a quarter, and four quarters make up a whole. Therefore, the denominator is crucial in comprehending that fractions express the relationship between parts and the whole, rather than solely focusing on the number of parts they possess. When students mistakenly believe that the denominator indicates the number of parts they have or equates to the numerator, they fail to grasp the fundamental structure of fractions.

Figure 1 in the grade 2 textbook illustrates the concept of $\frac{1}{2}$ (half) through two methods: dividing chocolate into two equal parts and visualizing flat shapes divided into two parts. At the bottom of the page, a contextual example is provided, such as dividing eight cakes into two flavors: four chocolate flavors and four strawberry flavors. “many chocolate cakes are half of 8 cakes, which is 4 cakes. Written, $\frac{1}{2}$ of 8 cakes, which is 4 cakes.”

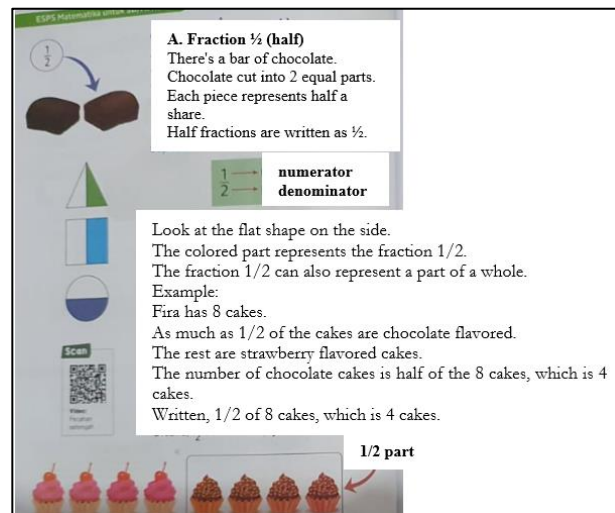


Figure 1. Introduction to fractions in the grade 2 elementary school mathematics textbook

Although this representation aims to facilitate understanding, this type of presentation can actually reinforce the misconception that fractions are the result of dividing numbers, specifically 8 divided by 2 equals 4. Students who encounter the statement “ $\frac{1}{2}$ of 8 = 4” without comprehending that “ $\frac{1}{2}$ ” signifies a proportion of the whole may mistakenly perceive that $\frac{1}{2}$ and 1:2 (division) are equivalent. Furthermore, in the practice section on the subsequent page, several images depict fractions through pizza slices or flat shapes that are partially shaded. However, not all visuals depict a whole that is divided evenly, and some images may be misinterpreted by students for various reasons. The visual representations lack explicit indications of equal partitioning, which is crucial for defining fractions. Without clear measurement of equal parts, students may interpret the division arbitrarily rather than proportionally. For instance, not all images represent consistent division, there is no explicit emphasis on the role of the numerator and denominator, and the part-whole relationship. This potential for misconception arises when textbooks present fractions as an activity of dividing and counting parts, without providing a conceptual explanation of the numerical meaning of the fraction itself. Such representations tend to be procedural and concrete, while students also require guidance to develop an abstract understanding of fractions (Björklund et al., 2020). From a praxeological perspective, the textbook predominantly emphasizes techniques (τ), such as dividing objects into equal parts, without sufficiently developing the underlying technology (θ), namely the conceptual explanation of fractions as part-whole relationships. Consequently, students develop procedural knowledge without theoretical grounding, leading to misconceptions when encountering abstract representations.

Fractions and Pronunciation (Question 2)

In most educational settings, students are taught to express fractions in their simplest form, such as “one by two,” “three by four,” or “two by three.” However, a notable group of 11 students (20%) opts to express fractions as “one divided by two” or “three divided by four.” This observation suggests that some students comprehend fractions not as components of a whole but rather as the outcome of division operations.

This misconception aligns with the findings of (Watt et al., 2025), which suggests that elementary school students tend to associate fractions directly with the form of division operations due to the use of language and visual representations that do not emphasize the concept of part of the whole. In the theory of cognitive development, Piaget also emphasized that elementary school children are still at the concrete operational stage, where understanding symbols such as $\frac{1}{2}$ requires assistance through concrete representations and appropriate conceptual language.

Interviews conducted with two second-grade teachers who participated in this study revealed that teachers were not fully cognizant of the potential for language such as “one divided by two”

to prompt students to interpret fractions as operations. One teacher expressed, “*I believe that ‘one divided by two’ aids children in visualizing concepts. I had never considered that it could lead them to misunderstand fractions.*” This statement underscores the significance of the lack of awareness regarding the impact of instructional language as a risk factor for the development of misconceptions. This aligns with the research conducted by Turan et al. (2026), which emphasizes that language in mathematics learning serves not only as a communication tool but also as a thinking tool. Consequently, improper use of language can result in erroneous comprehension.

Figure 2 depicts a page from a mathematics textbook (the page featuring illustrations of a watermelon, pizza, and apple) illustrating the introduction of fractions through concrete narratives, such as dividing fruit or food into multiple portions for sharing. For instance, in the watermelon illustration (Problem 1), it is stated that one watermelon is divided into three parts for three children. However, there is no explicit emphasis on the fact that $\frac{1}{3}$ constitutes a part of the whole; instead, the context focuses solely on the equitable distribution of the food among the children. Similarly, in the apple problem presented below, Siti divides the apple into two equal parts and consumes one part with her grandmother. Although fractions are visually represented, the accompanying narrative fails to elucidate the mathematical significance of the $\frac{1}{2}$ symbol. Narratives of this nature can lead students to erroneously conclude that $\frac{1}{2}$ signifies “divided in half,” rather than “one of two parts”—a distinction of paramount importance in the realm of mathematical concepts.

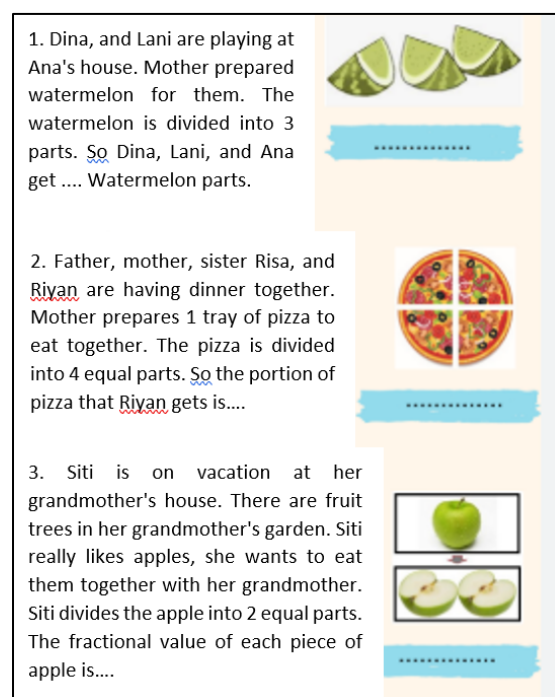


Figure 2. Illustrating fractions through contextual narrative language in the grade 2 elementary school mathematics textbook

This finding reinforces the arguments presented by Lamon (2007) and Petit et al. (2010) that visual representations devoid of accompanying conceptual and explicit language can exacerbate misconceptions, particularly in the domain of fractions. Fractions are inherently abstract, requiring a comprehensive understanding of the relationship between parts and the whole.

Contextual Representation of Fractions (Question 3)

Out of the 55 students, only 28 students (51%) correctly answered the question, which was $\frac{1}{4}$. In contrast, 16 students (29%) wrote “1:4,” and 11 students (20%) wrote “1 divided by 4.” These answers suggest that nearly half of the students tend to equate fractions with explicit division rather than understanding them as representations of parts of a whole. Several students who were

interviewed provided reasons for their answers, such as: “Ali got 1:4 because it was divided by 4 people”; “One divided by four = 1:4”; and “Because Ali only got one, then 1 out of 4 = 1:4.” As illustrated in the following interview excerpt:

Researcher: *"If you write 1:4, what does that mean?"*

S1: *"That's because it was divided by 4, so it's written as 1:4."*

S2: *"1:4 is one divided by four, so it's the same as a fraction, Ma'am."*

Interviews with several students confirmed that this understanding was not merely a writing error but a conceptual belief. This finding aligns with (Isnawan et al., 2022), who argue that students construct the meaning of fractions based on their interpretative experiences, which are influenced by instructional practices and representations. When these experiences are predominantly characterized by procedural interpretations such as division, students tend to internalize fractions as operational outcomes rather than conceptual relationships. Although students comprehend that the cake is divided into four parts and Ali receives one part, they express it in the form of a division operation (1:4), not a fraction symbol ($\frac{1}{4}$). From this, it is evident that the division and fraction symbols are perceived as equivalent, even by students who have frequently encountered fractions in visual form. This suggests a conceptual error in representing fractions, as students fail to grasp that $\frac{1}{4}$ is a form of rational number that signifies a portion of a whole, not an arithmetic operation. Conversely, the results of interviews with teachers revealed that the teaching approach they employed did not explicitly differentiate between fraction and division symbols. One teacher stated, *"I typically explain to the children that fractions are equivalent to division. For instance, one divided by two is half. They comprehend this concept more readily in this manner."* This statement indicates that the use of language and procedural approaches by teachers also reinforces the misconception that fractions are the result of division operations, rather than rational representations.

Visual media, such as the one depicted in Figure 3, which illustrates fractions through the use of concrete objects like apples, tomatoes, and halved boiled eggs, initially appears to be beneficial. However, upon examining the accompanying narratives found in textbooks and educational images, it becomes evident that students are only encouraged to “divide” objects into two, three, or four parts without being explicitly informed that each part constitutes a portion of a whole and possesses a fraction designation (e.g., “a quarter”). For instance, in Figure 3, the statement *“The apple is cut into 4 parts”* is provided without any subsequent explanation, such as *“Each part is referred to as a quarter of the entire apple.”* According to Aldianisa et al. (2021), the utilization of concrete contexts without emphasizing the mathematical significance of fractions can enhance procedural comprehension while simultaneously hindering the development of students’ conceptual understanding. Similarly, Watt et al. (2025) demonstrated that visual representations, such as food or tangible objects, necessitate explicit labeling of fractions to prevent students from misconstruing the division process as the outcome of the division operation.

This finding underscores the necessity of striking a balance between the utilization of concrete objects in fraction learning and the provision of introductory language that explicitly elucidates the concept of fractions as constituent parts of a whole, rather than merely as the outcome of division. Instructors should refrain from employing expressions such as “one divided by four” without accompanying clarification, as this inadvertently reinforces the association of fractions with arithmetic division operations, rather than as rational representations. Furthermore, instructional materials and textbooks should incorporate explicit mathematical labeling, such as $\frac{1}{4}$ or $\frac{1}{3}$, within every visual representation employed. This ensures that students can discern the correlation between the resulting parts and the entirety of the object being divided.

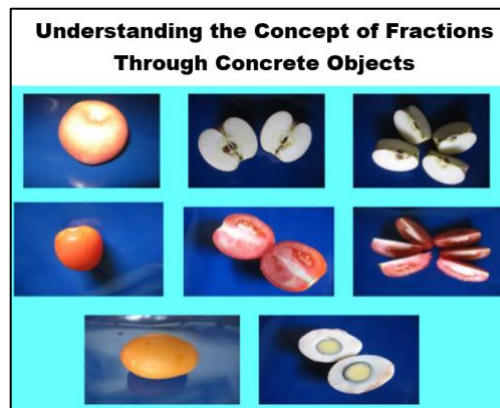


Figure 3. Illustrative example of introducing fractions via concrete objects in textbooks

Comparison of Fractions (Question 4)

The finding that only 19 out of 55 students (35%) answered correctly when asked to compare the fractions $\frac{1}{4}$ and $\frac{1}{2}$ confirms that conceptual difficulties in understanding fractional sizes are still very high. As many as 45% of students answered "same amount" or "don't know", and most students concluded that $\frac{1}{4}$ is greater than $\frac{1}{2}$ simply because the number 4 is greater than 2. This shows that students do not yet understand that in unit fractions (with a numerator of 1), the larger the denominator, the smaller the value. This misunderstanding can be further traced through the analysis of questions in the textbook, as seen in questions 2 and 3 in Figure 4. Both questions have the potential to strengthen students' misconceptions because they are not designed by considering the concept of proportion that makes sense contextually and logically. In question 2, students are asked to determine the number of kilograms of oranges bought by Father using fractions $\frac{4}{3}$, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{2}{3}$. The main problem in this question is the inconsistency of the context with the size of the fraction. The fraction $\frac{4}{3}$ is an improper fraction (greater than 1), but there is no narrative or visual that helps students understand that $\frac{4}{3}$ is greater than 1. Most students who are unfamiliar with improper fractions will assume $\frac{4}{3}$ is smaller than $\frac{2}{3}$ or $\frac{3}{4}$, because it has a larger denominator. This reinforces the misconception that fractions with larger denominators are smaller, regardless of the value of the numerator (Andriatna, 2025).

Below are presented story questions and answer choices listed on the fruit picture. Move the fruit with the correct answer to the basket provided!

- Andien takes an apple on the table. Then Andien cuts the apple into 5 equal parts. Andien only takes 2 parts. The rest, Andien will give to Arien. How many parts of the apple did Arien receive if expressed in fractional form?

$\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

- Father will take Adit to buy oranges at the market. Father chooses oranges because oranges are fruits that contain a lot of Vitamin C. Vitamin C is very good for our bodies, one of which is preventing mouth ulcers. Father and Adit buy $\frac{1}{4}$ kg of oranges. On the way home, Father and Adit meet an uncle. Uncle gives Adit $\frac{1}{4}$ kg of oranges. How many kilograms of oranges does Adit have now?

$\frac{1}{4}$ $\frac{3}{4}$ $\frac{2}{4}$ $\frac{1}{4}$

- Mr. Rahman will harvest avocados in his garden. After weighing, the total weight of the avocados is only $\frac{7}{8}$ kg. However, after being checked again by Mr. Rahman, it turns out that there are rotten avocados weighing $\frac{2}{8}$ kg. How much does Mr. Rahman's intact avocado weigh?

$\frac{4}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{3}{8}$

Figure 4. Fraction questions in textbooks

Problem 3 presents a very similar situation where students are asked to choose the largest fraction, but there is no visual or contextual explanation of what is meant by “the most intact avocado.” The word “most intact” intuitively refers to the largest number, but students who are unfamiliar with fraction comparisons often make mistakes in determining which of $\frac{3}{4}$, $\frac{2}{5}$, $\frac{4}{6}$, and $\frac{1}{2}$ is greater. They can be fooled by large denominators, such as thinking that $\frac{2}{5}$ is greater than $\frac{3}{4}$ because $5 > 4$, or because the numerator is small ($2 < 3$), so it is associated with a small value, without considering the proportional relationship.

The results of these two questions reflect students' misconceptions in comparing fractions, as also found in the main test questions. In interviews, students who answered incorrectly generally did not mention proportional reasons, but only said "this number is greater", "because 4 is greater than 2", or "because 3 is greater than 1". This shows that their understanding of fractions is still at the number level, not at the concept of parts of a whole or proportion.

As stated by Lamon, (2007), this kind of misconception arises because students do not yet understand that the denominator tells how many parts make up a whole, and the numerator tells how many parts are owned. When students are not taught to visualize fractions or compare with concrete aids, they only rely on incorrect numerical intuition. Manipulative approaches, such as cutting paper into 2 and 4 pieces and comparing their sizes directly, are essential for building conceptual connections (Van de Walle et al., 2020). Once students understand that the more pieces needed to make a whole, the smaller the size of each piece, they will be able to logically conclude that $\frac{1}{4} < \frac{1}{2}$, even though $4 > 2$ (Petit et al., 2022).

Understanding Fraction Symbols and Division (Question 5)

Of the 55 students, the majority (42%) chose the answer $\frac{4}{12}$, which mathematically accurately describes the fraction of the total books that Ali received. However, an important finding emerged from the option 1:3, which was chosen by 40% of students. This option shows that students tend to associate the context of “sharing” with the division symbol rather than fractions. In follow-up interviews, several students who chose 1:3 explained that “Ali got one of three types of books”, or “divided by three because there are three colors”, indicating that they used the number of types of books, not the total number of books, as the denominator of the fraction. This indicates a conceptual misconception, namely that students do not yet understand that fractions represent a part of a whole quantity, not the number of categories or types.

Furthermore, students who chose $\frac{1}{3}$ (11%) generally guessed, and were unable to explain exactly what $\frac{1}{3}$ meant in that context. Meanwhile, 4 students (7%) who answered 12:4 tended to treat the fraction as a direct division operation, namely “12 divided by 4”, when what was asked was the proportion of the part that Ali received. These results correlate with the content analysis of the textbooks used, where fraction representations are mostly introduced in the form of narratives of dividing objects, such as “6 apples were divided among 3 children”, or “some objects were grouped evenly”. These representations, although concrete and contextual, underemphasize the structure of fractions as part of a whole unit (Watt et al., 2025; Lamon, 2007). As a result, students are accustomed to associating fractions with the process of “being divided”, rather than as “parts of a whole”.

Two illustrations from the analyzed textbooks support this finding. Figure 5 shows that out of 12 notebooks, 3 books colored blue represent $\frac{1}{4}$ of the whole. However, the way it is presented by grouping and coloring some of the books (3 out of 12) visually resembles the process of “dividing into four parts”, which can also be interpreted as the result of dividing 12 by 4.

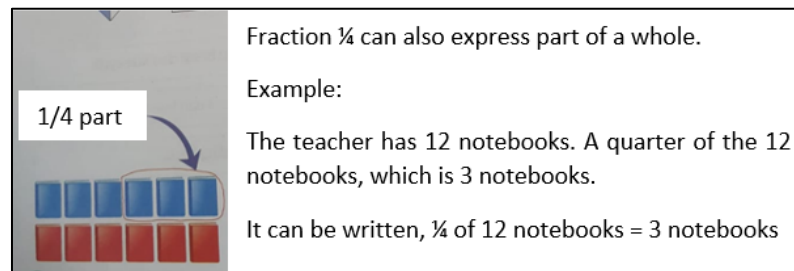


Figure 5. Illustration of fractions as part of a whole in textbooks

Figure 6 illustrates Jajang distributing eight apples equally among his four friends, resulting in each child receiving two apples. This visual representation effectively demonstrates the division of 8 by 4. Despite the distinct contexts depicted in the two images (fractions and divisions), both employ a consistent visual strategy: grouping objects in a row to signify “part of the whole.” Consequently, students often struggle to discern that in one context (the book illustration), the concept represented is a fraction (part-whole relationship), whereas in the other context (the apple illustration), it pertains to a sharing division.

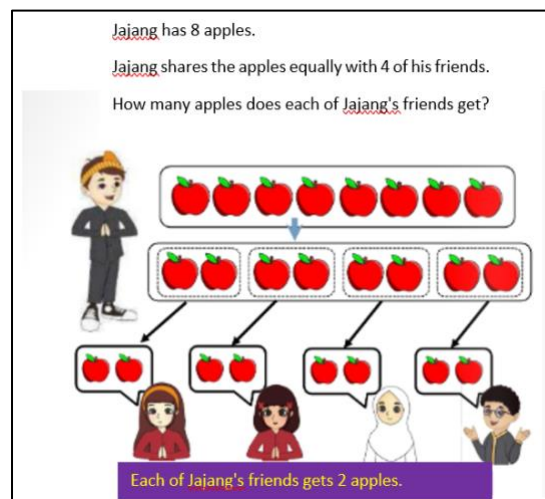


Figure 6. Visual representation of the concept of division in textbooks

The visual representation of fractions, often depicted as evenly grouped blocks of objects, can inadvertently reinforce the misconception that fractions solely represent the result of division. In interviews, several students who encountered the problem “4 out of 12 books” responded with the ratio 1:3, suggesting that they tend to interpret fraction situations as a division process rather than a proportion of a whole. Notably, the majority of the representations encountered in textbooks are procedural, lacking emphasis on the conceptual significance of fractions. This aligns with Lamon’s (2007) findings, which indicate that without diverse representations and explicit explanations of the meaning of fractions, students tend to develop narrow understandings based on their experience of “dividing things.” Consequently, it is crucial for fraction instruction in early grades to employ not only concrete representations but also to explicitly present the conceptual distinctions between fractions as a part-to-whole relationship and division as an arithmetic process. This approach can be further enhanced by utilizing multiple representations, such as area models, number lines, and set models (Björklund et al., 2020). These representations enable students to comprehend fractions from various perspectives, thereby mitigating the risk of being confined to a singular procedural viewpoint.

These findings underscore the necessity of employing a learning approach that incorporates multiple representations, including area models, set models, and number lines. These representations enable students to perceive fractions as components of a whole that are divided proportionally, rather than merely the outcome of a division operation (Björklund et al., 2020).

Additionally, educators should emphasize that fraction symbols, such as $4/12$, signify a part-to-whole relationship, not merely “divided” in the context of an arithmetic operation. By reinforcing these concepts through a diverse range of concrete and visual activities, students will be better equipped to comprehend fractions flexibly and meaningfully.

CONCLUSIONS

This study elucidates that one of the primary sources of elementary school students’ conceptual misconceptions in comprehending fractions resides in the manner in which the concept is introduced through open books and classroom learning practices. Visual and narrative representations in grade 2 mathematics textbooks that describe tend to equate fractions with division results (e.g., $\frac{1}{4} = 1:4$), thereby reinforcing procedural understanding and fostering the interpretation of fractions as a component of a whole. The findings of interviews and observations indicate that the majority of students interpret fractions in the context of division directly, without grasping the conceptual structure.

Theoretically, this study bolsters previous findings that concrete and procedural representations that are not balanced with contextual explanations can impede the development of abstract mathematical understanding. The novelty of this study lies in the conceptualization of textbook content analysis and classroom practices in an integrated manner to trace the origins of students’ misconceptions, which have not been comprehensively studied in the Indonesian context.

The implication of this finding is the necessity for a more balanced approach to fraction learning in early grades between concrete and abstract representations, as well as the utilization of multiple representations (area, length, set) that can enrich the manner in which students construct the meaning of fractions more flexibly. Furthermore, teachers need to be trained to identify potential misconceptions from open-ended materials and develop more conceptual teaching strategies. This study contributes to the theory of mathematics learning by emphasizing the significance of conceptual representation in early fraction learning to establish a robust and enduring foundation of understanding.

Future research should investigate intervention-based studies that design and implement instructional approaches integrating multiple representations and praxeological analysis. Additionally, further studies involving a larger and more diverse sample across various regions are required to generalize the findings and examine the impact of curriculum variations on students’ fraction understanding.

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