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Using 6 questions cognitive method and polya's model to helps students solve mathematical problem

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Abstract. The ability to solve the mathematical equation problem is very important for the thinking development of middle school students. "How to solve it" of George Polya is famous in the world. 6 questions cognitive model proposed by Professor Zhou are monitored through metacognition, and the 6 questions are coherent, complete and sequential. This paper found that the 6 questions cognitive model can help implement "How to solve it" of George Polya and reduce students' cognitive load. At the same time, this study found that 6 questions cognitive model can help students solve the mathematical equation problem better.

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INTRODUCTION

Mathematics subjects in each country emphasize that students should have a good ability to solve exercises, especially to solve the mathematical equation problem that combines real-life and mathematical knowledge (Hendriana, Rohaeti, & Hidayat, 2017; Kang, 2015). Solving problems can reflect the students ' problem-solving ability (Dewi, Khodijah, & Zanthy, 2020), analysis skill, logical thinking ability (Haj-Yahya, 2021), etc. Therefore, cultivating students to learn to solve mathematical equations is of great help to students' future development of mathematics. But at present, many students in the test paper often can't do mathematical equation problems (Fatio, Fatimah, & Rosjanuardi, 2020; Puspitasari, Yusmin, & Nursangaji, 2015; Wijaya, Ying, Chotimah, et al., 2020). They think it is difficult to analyze the logical relationship, resulting in them nervous and do not know how to solve them, so they lose their scores. Therefore, this article tries to use "6 questions cognitive model" to help students understand "how to solve it" of George Polya to solve the mathematical equation problem better.

As is well known, the "how to solve it" proposed by the famous mathematician George Polya provides appropriate steps for solving problems and provides teachers with ideas for problem-solving teaching (Haryanti, Herman, & Prabawanto, 2019; Hidayat & Sariningsih, 2018). It is divided into four steps: understanding a problem, formulating a plan, implementing a plan, and reviewing it (Hidayat & Sariningsih, 2018; Shirali, 2014). Each step leads the student to transform unknown problems into known problems through a series of sub problems. George Polya believes that the most important thing to solve a problem is finding the idea of solving problems (Hidayat & Sariningsih, 2018; Son, Darhim, & Fatimah, 2019). Thus, the process of

solving problems is a process of constantly exploring and inducing good ideas.

The 6 questions cognitive model proposed by Professor Ying Zhou is monitored by metacognition, including "where does knowledge come from?", "what is the essence of knowledge?", "what is knowledge related to?", "how to solve it?", "can it be solved in other methods?", "can you use this method to solve other questions?", "after learning this solution-solving method, what is the gains and reflection?", and the 6 questions cognitive model has coherence, integrity and sequence (Wijaya, Ying, Cunhua, & Zulfah, 2020; Yi, Ying, & Wijaya, 2019).

6 questions cognitive model is one of the learning models that help the systematic thinking process (Wijaya, Ying, Cunhua, et al., 2020). 6 questions cognitive model developed by Zhou Professor of 4mat Model (Lin, Zhou, Wang, & Wijaya, 2020). The development makes the 4mat models more effective and efficient for teaching (Cunhua, Ying, Qunzhuang, & Wijaya, 2019). Make the atmospheric learning more alive and students continue to concentrate during learning. 6 questions cognitive model reminds students to problem raising, development, solving problems, making the conclusion, deep learning processes, and connecting the knowledge that students have learned with students whom students will learn (Yi et al., 2019).

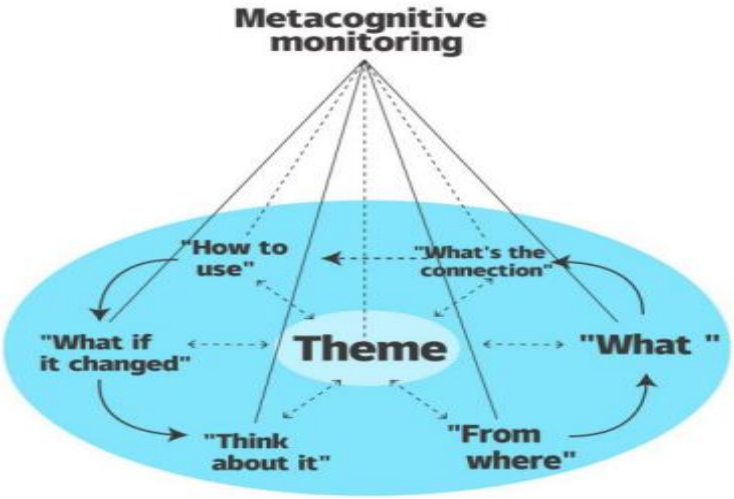

Figure 1. 6 questions cognitive model

Although "how to solve it" of George Polya is good, it usually feels not specific enough, the four big steps are too broad, and the sub-steps are too many, which are not easy to remember and use. The 6 questions cognitive model can help implement "how to solve it" of George Polya, "from where" and "what" corresponds to the first step "understanding a problem", "what`s the connection" corresponds to the second step "formulating a plan", and "how to solve" corresponds to the second and third steps "formulating a plan" and "implementing a plan", "what if it changed" and "think about it" corresponds to the fourth step "reviewing it". The details are presented in Figure 2.

In this stage of "understanding a problem", students are required to find out what the unknown quantity, known quantity and condition, that is the origin and background of knowledge ("from where" of the 6 questions cognitive model). Then express the text, symbols, graphics and other information in the question through the list, drawing and other ways, you can clarify the question type and problem and lay the foundation for finding the idea of solving problems. ("what" of the 6 questions cognitive model).

Formulating a plan is the key link in solving problems. We should find out the relationship between known data and unknown quantities. For familiar questions, we will have a fixed problem-solving routine, and for unfamiliar questions, if you can`t find a direct contact, consider the auxiliary problem ("what`s the connection" of the 6 questions cognitive model).

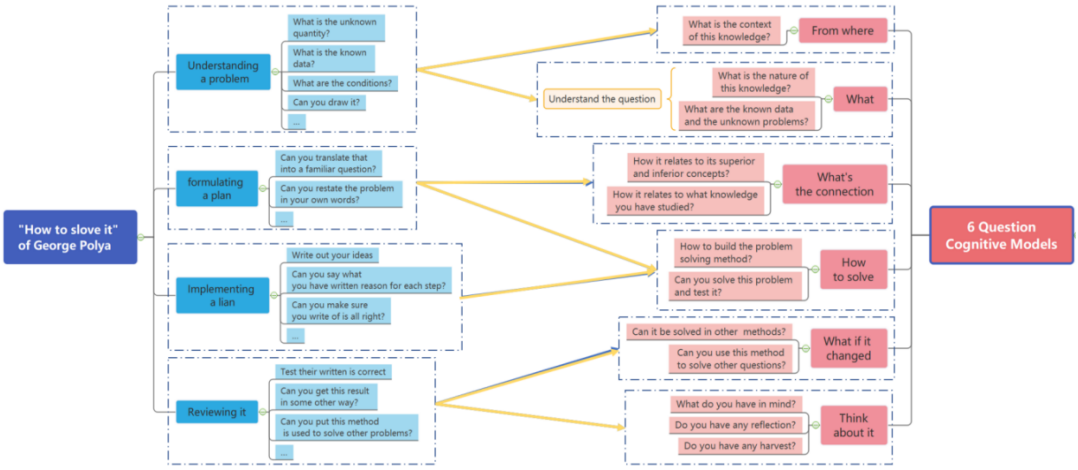

Figure 2. The connection between 6 questions cognitive model and "how to solve it" of George Polya

In the "implementing a plan", the problem solver needs to use Mathematical specification format to describe the process of solving the problem, and step by step reasonable, step by step evidence ("how to solve" of the 6 questions cognitive model). Therefore, in this step we need to ensure the correctness of each step.

At "reviewing it" stage, we should not only test whether the calculation answer is correct, but also consider what ideas and methods are drawn from it. ("think about it" of the 6 questions cognitive model). And then, we think about whether we can try a problem with multiple solutions, whether we can use this solution corresponding to multiple problems ("what if it changed" of the 6 questions cognitive model).

METHOD

This study discusses 6 questions cognitive model to teach students how to solve the linear equations in two variables. The first step is a design study. Two Chinese students, one international student and Professor Zhou from at Guangxi Normal University, discussed the research's significance and the previous research results. The second step is to determine the innovation points, methods, and objects of the research. The third step, collect the experimental results and analyze the research. The flowchart is shown in Figure 3.

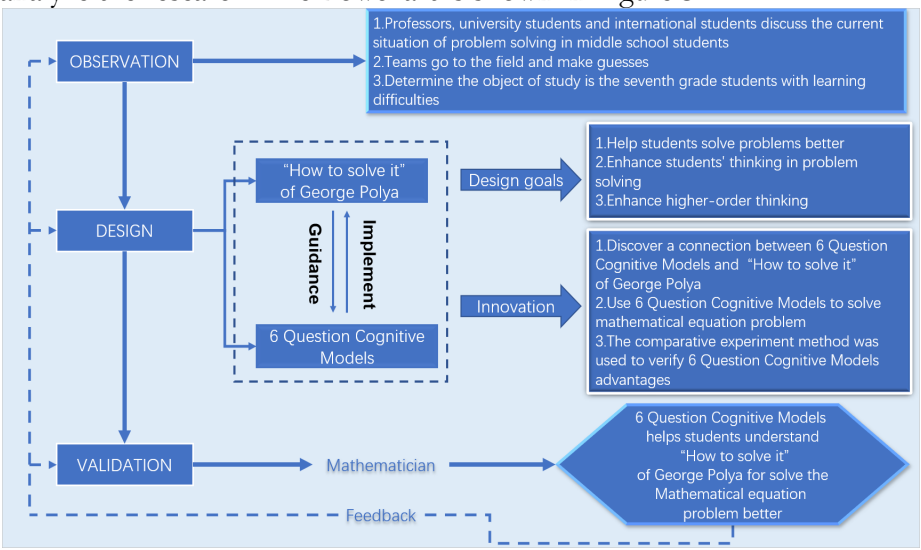

Figure 3. Research planning and purpose

RESULTS AND DISCUSSION

The teaching objectives require that the students learn to analyze and solve the life situation problems, and they can use the knowledge of the linear equations in two variables to solve the practical problems. This process improves students' ability to analyze and solve problems and experience transforming ideas and equations in the application and improvement. To this end, a class was designed, the classroom used six what to teach students to solve the problem, and the comparison method to prove the effect of 6 questions cognitive model.

Here we teach students to analyze and solve some mathematical equation problems by 6 Questions Cognitive Model to help students understand the topic, formulate plans, implement plans and review, and improve their ability to analyze and solve problems.

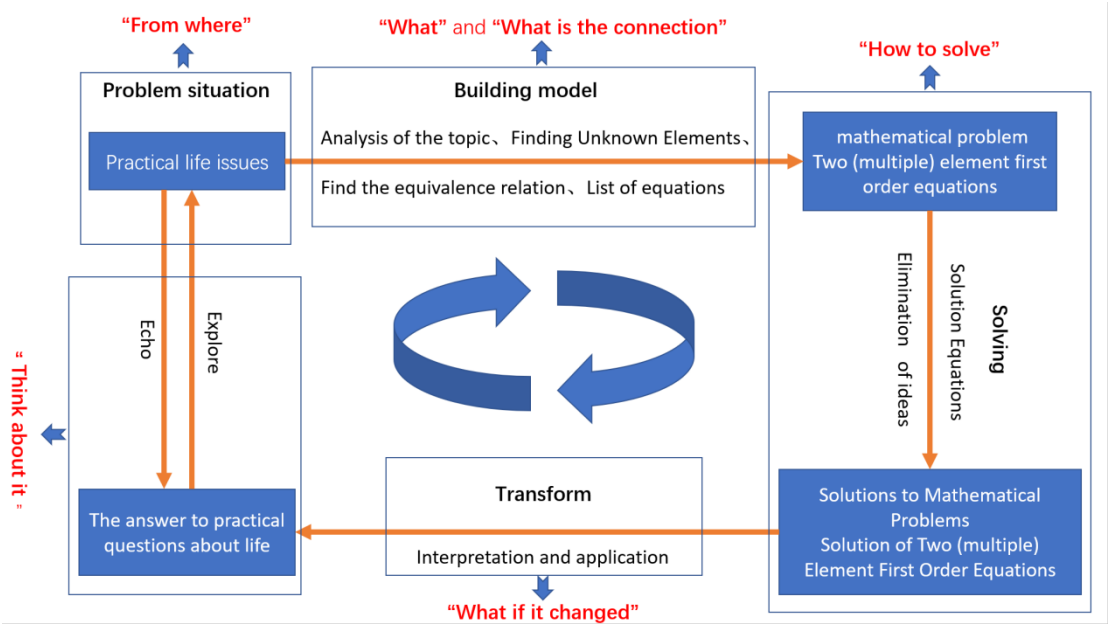

Figure 4. Process of solve problems by 6 questions cognitive model

Q: A city stipulates that the taxi starting price includes a distance of 0-3 km, more than 3 km at a separate charge per kilometer. A said: "I took this taxi 11 km, paid 17 yuan". B said: "I took this taxi 23 km, paid 35 yuan ". Please calculate: how much is the starting price of a taxi? After more than 3 km, how much is the fare per kilometer?

understands the topic and feels "From where"

"From where" is to trace the source, find the connection between the situation problems and the mathematical problems, and then further discover the knowledge points and methods related to the mathematical problems. Students must find out unknown quantities, known quantities and conditions, that is, to understand the context of knowledge.

Teacher: The above question is an actual life scene. What can we think of from this question? What words and sentences will you focus on? What information is grab?

Student: This is the scene that we usually go out, from this problem I can think of time, speed, distance and unit price, quantity, total price, while I focus on the topic has two problems, this let me think of setting up two unknown or set an unknown, with a formula represents another unknown, I also thought of 3km is a fixed price, 3km per kilometer above another unit price, and gave us the total journey and total price, perhaps I can solve the unit price quantity total price relationship.

understands the essence and analyzes "what"

"What" means to find out the essence of the problem, accurately grasp what you need in

many information, through the list, drawing and other ways of the text in the problem, symbols, graphics and other information provided, that is, clear the problem type and problem, to lay the foundation for the search of solving ideas.

Teacher: Can we use the math language or the graphic language to describe the topic? What do you think is the key to solving this question?

Student: We can first draw a line section to represent the total journey, divided into two sections, one for 0-3km, and the other for more than 3km. The key to solving this problem is that different distances correspond to different prices respectively, so we need to grasp the relationship between the number of unit prices and the total price.

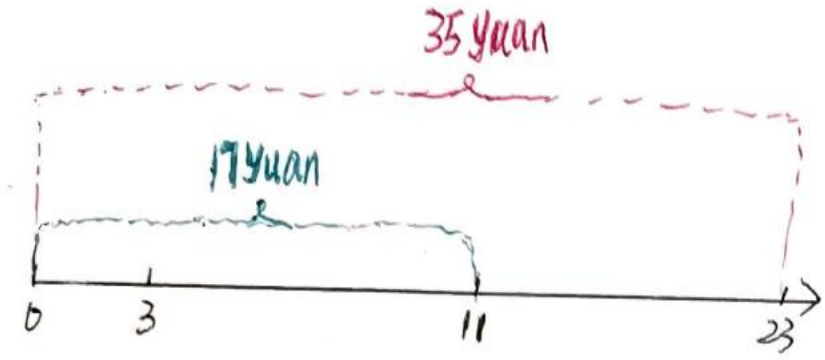

Figure 5. Distance map to help students understand the topic

develops plans to explore "What`s the connection"

The plan is the key link in Polya's solving problem. We explore "What`s the connection" and ask whether we have ever seen similar questions, similar differences with similar topics. What knowledge points are connected with, and can we think of a used theorem or formula? Are all the conditions used?

Teacher: What are the general steps of solving the application questions? From our known conditions and drawings, what isometric relationship can we establish?

Students: General steps of solving application questions: Review questions--set unknown number--Establish equequation--solve an equation--check-up--answer, from the known we can establish two equal weight relations, over 3km journey * Price over 3km + Price within 3km = total price, that is, the starting price of a taxi is x yuan, over 3km, the fare per kilometer is y yuan, listed as

$$(11-3)y+x=17 \text{ and } (23-3)y+x=35$$

Execution Plan, implementing "With What have Connection"

When implementing the solution solving plan, the solution solving person needs to use professional terms to express the solution solving process and step by step justified. Therefore, in this step, we need to ensure the correctness of each step.

Teacher: Can you complete the whole problem-solving process?

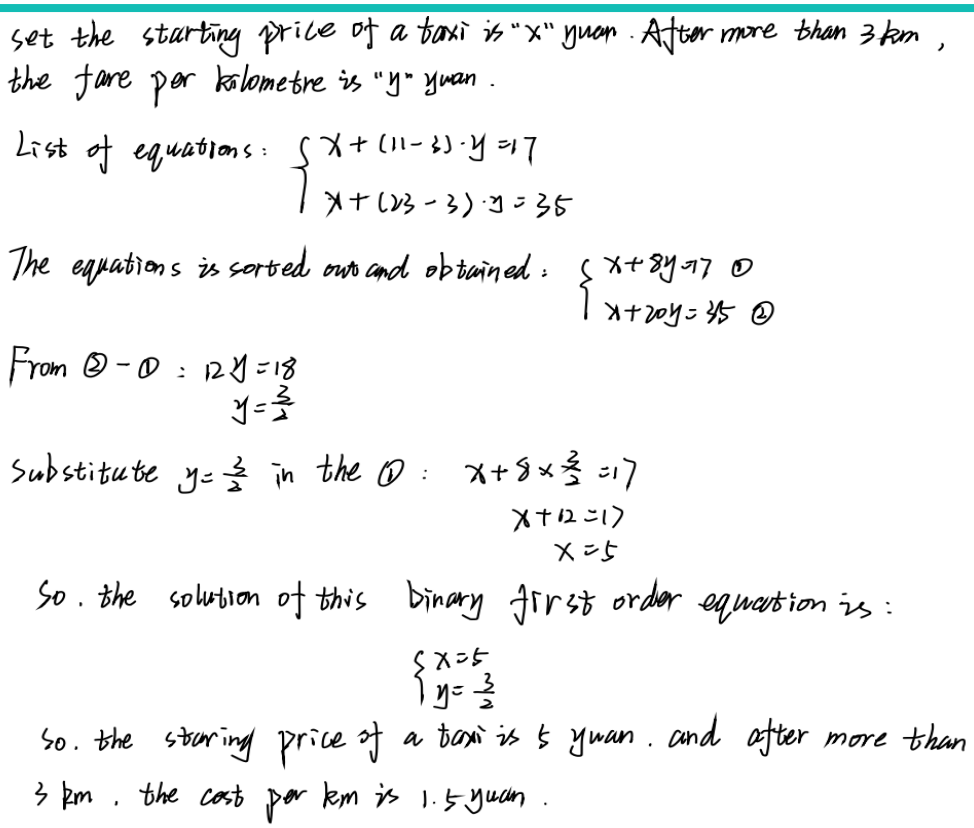

Figure 6. The problem-solving thinking of 6 questions cognitive model

Solve problems flexibly and develop the ability to "What if it changed"

"What if it changed" is a variant expansion, it can improve the vastness and flexibility of students 'thinking and develop students' divergent thinking, mainly including a multiple solution and a changeable, reverse thinking.

Teacher: Can you change the topic for a condition to form a new topic?

Student: Maybe I can change the starting price from anonymous data to known data, and solve the journey.

growth experience, reflecting on "Think about it"

Exploration, and summary of the common characteristics. Only when we summarize a kind of questions, find out their commonalities and become the idea of solving problems, can we learn to solve the questions when they change in the future.

Teacher: What rules can you sum up through this question? Can the result or method of this problem be used for other problems?

Student: I think this is a mathematical equation problem, the key to solving the problem is to grasp the relationship between the number of unit price and the total price. Among these problems, there are often the speed, time and distance relationship, working efficiency, working hours and total work problems. Finally, we should also be familiar with solving mathematical equations, accurately setting the unknown, and finding an equal relationship.

growth experience, reflecting on "Think about it"

According to the students just proposed in the "what if it changed" idea, adapt the topic into the following questions for the students to solve:

A city regulation: taxi starting price is 3 yuan, a said:" I took this kind of taxi walked 11 km, paid 17 yuan ". B said:" I took this taxi 23 km, paid 35 yuan ". Can you calculate the distance included in the starting price of this taxi? How much is the extra charge per kilometer over this

starting distance? Please take the group as the unit to discuss, send the representative to explain on the stage.

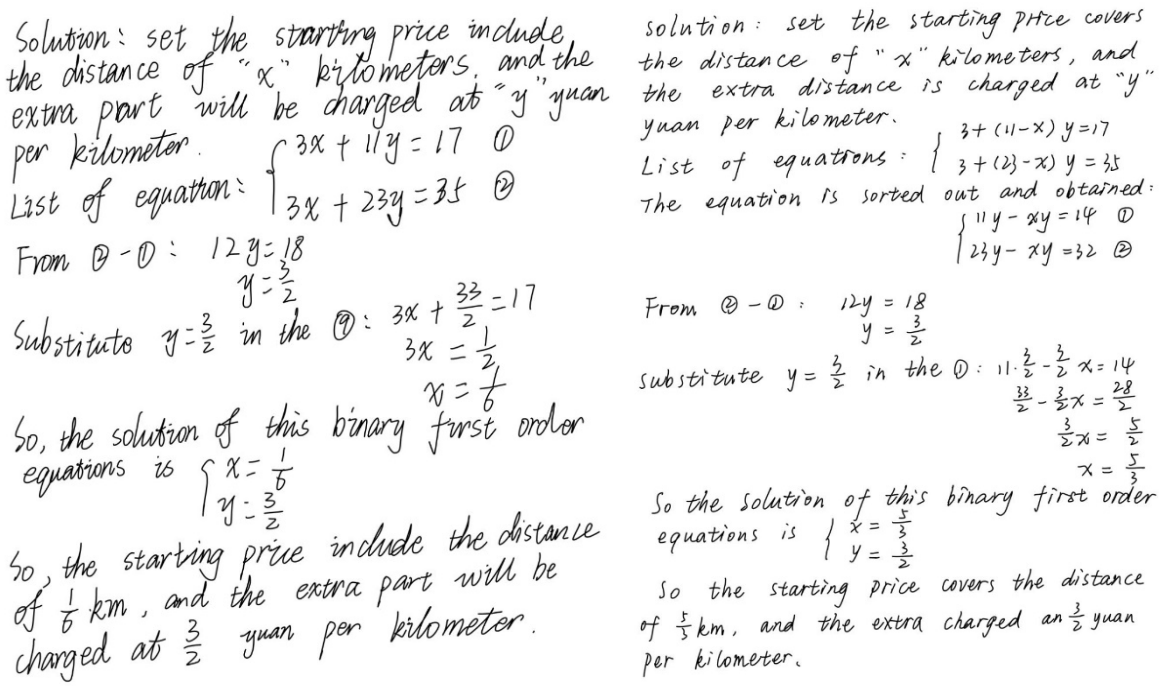

Figure 7. Answer from the first student and the second student

The two students wrote different solution processes and answers, in which Student 1 was wrong and Student 2 was correct.

We analyzed the problem-solving process of the two students, found that the student 1 did not understand the meaning of "starting price" at the beginning, set the unknown, did not think on their own, did not understand what x and y mean in the topic exactly, leading to the writing of 3x and 11y. In the end, the student 1 directly wrote the answer, did not consider whether the answer is correct, whether in line with the reality, nor did not summarize the rules and key of the question.

In contrast with Student 1, Student 2, Student 2 first understood the topic, found the connection between the situation problem and the mathematical problem, correctly found the unknown quantity, known quantity and conditions, grasped the key to solving the problem, and correctly identified the problem formulated the problem-solving plan. We found that Student 2 could associate the previous variant and clarify the plan through the relationship between distance and unit price. When implementing the plan, the students accurately set up the unknown number, list the system of equations, and use the binary primary equation system solution method to calculate the answer accurately, each step is justified. Finally, the student also reviewed the problem-solving problem, where the student tested the correctness of the calculation.

CONCLUSIONS

The above analysis found that the student 2 solution idea was completely in line with 6 Questions cognitive Theory, and he understood the meaning of the starting price. On the other hand, student 1 pursues to solve problems quickly, not doing well in "From where" "what" "What`s the connection" "how to solve", so I did the wrong topic. Therefore, if we can understand the topic, formulate the plan, implement it and review the "6 Questions Cognitive Model" idea, we will be able to solve it correctly.

First, "How to solve it" of George Polya and the 6 Questions Cognitive Model are not only applied to a slightly complex "difficult problem", but through the whole process of education; Secondly, 6 Questions Cognitive Model is not only applicable to mathematical equations, but also to other mathematical content; Finally, the 6 Questions Cognitive Model is not only applicable to problem-solving exercises but also applicable to principles classes, concept classes, review classes and so on.

I hope that more and more scholars will study 6 Questions Cognitive Model in the future, strive to cultivate students 'logical thinking, improve their high-order thinking, and learn to solve problems better.

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